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AVERAGING OF PERIODIC PRESSURE PULSATIONS BY

A TOTAL-PRESSURE PROBE

By R. C. Johnson

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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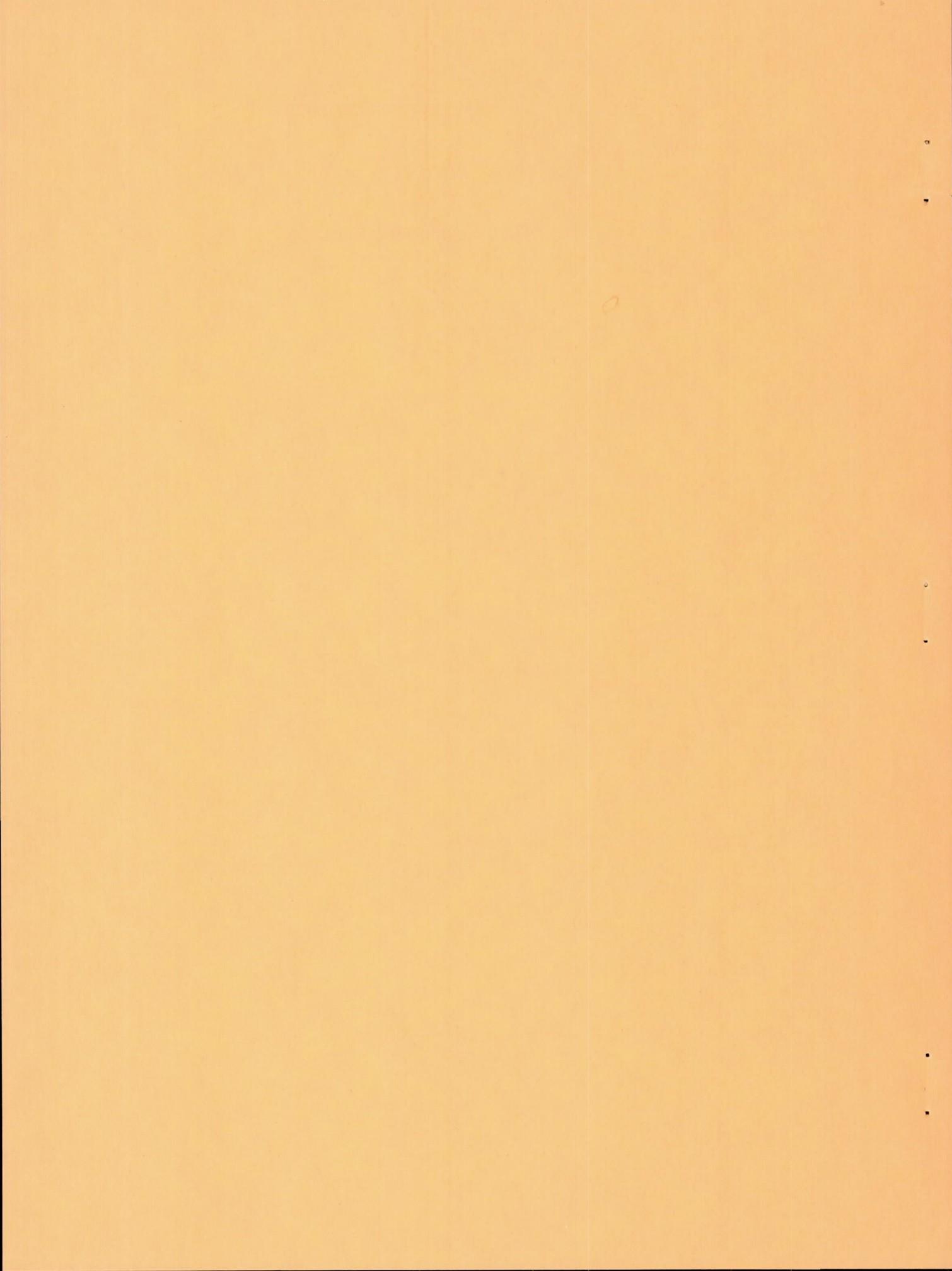
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SUMMARY

Information is presented on the average pressure indicated by a total-pressure probe subjected to a stagnation pressure that alternates periodically between two constant values.

Calculated and experimental data are in good agreement, and errors are reduced when the probe design is such as to ensure laminar-flow pulsations in the probe at all times. The averaging error is minimized when the inside diameter of the probe entrance tube is made as small as possible, and its length as great as possible, consistent with an acceptable time lag.

INTRODUCTION

When a total-pressure probe, connected to a manometer, is subjected to a fluctuating pressure, the manometer indication may not represent the time-weighted average of the applied total pressure because of certain nonlinear effects. When the wave length of the pressure fluctuation is large compared with the probe length, and the period of the fluctuation is small compared with the time constant of the probe and its associated pneumatic system, the principal nonlinearities are reduced to two.

For a total-pressure probe whose entrance tube is so long that its end effects can be neglected, the mass-flow rate of incompressible laminar flow within the tube is a linear function of the applied pressure. Under these conditions the probe will indicate the time-weighted average of the applied pressure. For compressible laminar flow in the entrance tube, the mass-flow rate is also a function of the average density within the entrance tube. Since this average density is, in turn, a function of pressure, the mass-flow rate then becomes a nonlinear function of the applied pressure. Thus, the probe will not give the time-weighted average of the applied pressure.

For a total-pressure probe whose entrance tube is so short that it can be considered an orifice, the mass-flow rate of incompressible flow through the orifice is proportional to the square root of the pressure drop across the orifice. Since this is a nonlinear function of the applied pressure, this probe will not give the time-weighted average of the applied pressure. If the flow is compressible, additional nonlinear effects will result.

A third possible nonlinearity results from a difference between the flow coefficients for the flow in and out of the probe. Because the restricted part of the probe is essentially similar with respect to the flow processes both in and out of the probe, this particular nonlinearity is not believed to be significant for the probe geometries considered in this report.

In general, the entrance tube of the total-pressure probe will be such that both principal nonlinear effects will be present. These effects are interdependent, and they must be considered simultaneously.

In reference 1 the averaging of some symmetrical wave shapes is analytically treated. This report will treat the averaging of pressure fluctuations of rectangular wave shapes (fig. 1). The averaging errors are first estimated analytically. Then the results of experiments on some typical probes are used to establish the probe design limits within which the theory may provide an adequate estimate of these errors.

ANALYSIS

The probe shown in figure 2(a) is a tube of length L and inside diameter D that opens into another tube of much larger diameter. The latter tube connects the probe to a manometer, so that the pressure at the exit of the probe is the same as the pressure at the manometer. The pressure P at the probe entrance is a stagnation pressure that follows the periodic pattern shown in figure 1.

The time-weighted average of the applied pressure P is

$$P_a = P_{min} + \beta(P_{max} - P_{min}) \tag{1}$$

The probe-averaged pressure P_m can be represented as

$$P_m = P_{min} + \alpha(P_{max} - P_{min}) \tag{2}$$

Combining equations (1) and (2) yields

$$P_a = P_m - (\alpha - \beta)(P_{max} - P_{min}) \tag{3}$$

Thus $(\alpha - \beta)(P_{max} - P_{min})$ is the averaging error due to nonlinear effects. This error is to be subtracted algebraically from the manometer indication P_m to yield the time-weighted average pressure P_a.

At any instant, the pressure drop $|P - P_m|$ across the probe entrance tube is given by

$$|P - P_m| = (\Delta P)_1 + (\Delta P)_2 \tag{4}$$

where $(\Delta P)_1$ is the pressure drop due to the sudden contraction, and $(\Delta P)_2$ is the pressure drop due to friction within the tube. The static-pressure change across the sudden enlargement is negligible.

The pressure drop due to the sudden contraction may be estimated from the orifice-flow formula. The relation between pressure drop and mass-flow rate per unit area G is given by

$$G = C\left\{\frac{2\gamma}{\gamma - 1}\frac{p_b^2}{R\theta}\left(\frac{p_c}{p_b}\right)^{\frac{2}{\gamma}}\left[1 - \left(\frac{p_c}{p_b}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \tag{5}$$

where p_b and p_c are the pressures upstream and downstream of the tube entrance, respectively. For subcritical pressure ratio, equation (5) may be approximated to within 3 percent by

$$G = \frac{C}{\sqrt{R\theta}} \left[2p_c(p_b - p_c) \right]^{1/2} \tag{6}$$

The approximate relation between pressure drop and mass-flow rate through the tube is

$$G\sqrt{f} = \left[\frac{D}{4L} \frac{1}{R\theta} (p_c^2 - p_d^2) \right]^{1/2} \tag{7}$$

where p_d is the pressure at the downstream end of the tube and the friction factor f is

$$f = \frac{16}{Re} \quad \text{for laminar flow} \tag{8}$$

$$f = 0.046Re^{-0.2} \quad \text{for turbulent flow} \tag{9}$$

Applying equations (6) and (7) first to the case where the flow in figure 2 is from left to right, and then to the case where the flow is from right to left, and then introducing the condition that the total mass flow is zero over a complete cycle T give for laminar flow in the tube (see appendix B)

$$\beta^{-1} = 1 + (1 + \alpha\psi)^2 \frac{\delta_L}{\delta_R} \frac{2 + \delta_L}{2 + \delta_R} \tag{10}$$

where

$$\psi = \frac{P_{max} - P_{min}}{P_{min}} \tag{11}$$

$$\delta_L = \frac{-1 + \left[1 + 4\eta^2(1 + \alpha\psi)\psi(1 - \alpha)\right]^{1/2}}{2\eta^2(1 + \alpha\psi)^2} \tag{12}$$

$$\delta_R = \frac{-1 + (1 + 4\eta^2\alpha\psi)^{1/2}}{2\eta^2} \tag{13}$$

$$\eta = \frac{D^2 P_{min}}{32\mu LC \sqrt{2R\theta}} \tag{14}$$

When the pressure drop across the tube is large compared with the pressure drop due to the contraction,

$$\beta^{-1} = 1 + \frac{1 - \alpha}{\alpha} \frac{2 + (1 + \alpha)\psi}{2 + \alpha\psi} \tag{15}$$

When the pressure drop across the tube is small compared with the pressure drop due to the contraction,

$$\beta^{-1} = 1 + \left[\frac{1 - \alpha}{\alpha} (1 + \alpha\psi)\right]^{1/2} \tag{16}$$

For turbulent flow in the tube, the solution is algebraically quite complex, unless the pressure drop across the tube is either very large or very small compared with the pressure drop due to the contraction. If the pressure drop in the tube is much smaller, equation (16) again results. If the pressure drop in the tube is much larger, the expression for β becomes

$$\beta^{-1} = 1 + \left[\frac{1 - \alpha}{\alpha} \frac{2 + (1 + \alpha)\psi}{2 + \alpha\psi}\right]^{5/9} \tag{17}$$

The relation in equation (10) can be represented by new variables which may be more readily measured in a practical situation (appendix B).

Let

$$A = 1 - \frac{P_{min}}{P_m} \tag{18}$$

$$X = \frac{P_a - P_m}{P_m - P_{min}} \tag{19}$$

Note that

$$P_a = P_m(1 + AX) \tag{20}$$

Then

$$X = \frac{\beta}{A} \delta_L \left[1 + \frac{\eta^2}{(1 - A)^2} \delta_L\right] - (1 - \beta) \tag{21}$$

where

$$\delta_L = -1 + \left[1 + 2 \left(\frac{1 - \beta}{\beta} \right) (1 - A)^2 \delta_R (1 + \delta_R/2) \right]^{1/2} \qquad (22)$$

and

$$\delta_R = \frac{-1 + \left(1 + 4\eta^2 \frac{A}{1 - A} \right)^{1/2}}{2\eta^2} \qquad (23)$$

The value of X as a function of β, η, and A is listed in table I. An example of the use of this method is given in appendix C.

EXPERIMENTAL INVESTIGATION

A typical probe is shown in figure 3. The ends of the entrance tube were carefully squared and deburred. A spacer was soldered near the end of the entrance tube within the probe to ensure centering within the body of the probe. Twenty-two probes were constructed representing 13 different geometries. The characteristics of probes having the same geometry agreed closely.

The principal parts of the test apparatus are shown in figure 4. By means of a wheel and crank arm the probe was oscillated past the inner edge of a 0.75-inch-diameter nozzle forming a high-velocity air jet. The oscillation frequency could be varied from 3 to 50 cps. The pressure-wave shape was determined by rotating the wheel by hand and observing the pressure as a function of wheel angle. This wave shape was approximately rectangular at all jet velocities. Since, in all cases, the probe velocity was small compared with the jet velocity, the wave shape can be considered independent of frequency. In order to reduce pressure fluctuations behind the probe entrance tube at low frequencies, the probe was connected to a 3-cubic-inch volume by a length of 0.25-inch-inside-diameter plastic tubing. Probable errors of measurement were less than 1 percent in $P_m - P_{min}$ and less than 0.01 in β.

RESULTS AND DISCUSSION

In table II the characteristics of probes representing 13 different geometries are tabulated. In figure 5 a graphical comparison of theoretical and experimental probe errors is shown for six of the probes. In

these curves, $\alpha - \beta$ is plotted against β with ψ as the curve parameter. Figures 5(a) to (d) indicate generally good agreement with theory for probes whose geometry varies from one (probe 1, fig. 5(a)) whose entrance tube is such that it acts like a long capillary to one (probe 6, fig. 5(d)) that has a 0.020-inch orifice at the end of the probe. Probe 10 (fig. 5(e)) agrees with theory for low values of ψ. At high values of ψ, the agreement is poor. The agreement of probe 13 (fig. 5(f)) with theory is generally poor.

In this experiment the independent variables were β, η, and ψ. The dependent variable was $\alpha - \beta$. It should be noted that, in the expected application of this theory, the quantities that could be estimated in advance would generally be β, η, and A. From these, the time-weighted average pressure can be determined.

Four limitations exist which, if violated, would seriously affect the predictability of probe behavior:

(1) The entrance tube should open into a tube of much larger diameter (4 to 8 times, e.g.).

(2) The flow across the ends of the tube should always be subcritical.

(3) The period of fluctuation T should be much smaller than the time constant τ_0 of the probe and associated system and much larger than the time τ_1 required for a pressure disturbance to be propagated along the probe entrance tube. If the speed of the pressure disturbance is taken as the speed of sound under ambient conditions, then

$$\tau_1 = L/a \tag{24}$$

The preceding limitations can be expressed by the following inequality:

$$2\pi\tau_0 >> T >> L/a \tag{25}$$

(4) The flow within the probe should be either laminar over a complete cycle or turbulent over a complete cycle. It should be noted that it is possible to have the flow laminar over one part of a cycle and turbulent over the other part of a cycle. This situation does not readily permit analytical treatment.

Over the range of parameters and geometries covered in this investigation, it was found that the agreement between theory and experiment was

(1) Good when the maximum Reynolds number in the tube was less than 8,000. In this case, the laminar-flow equations (10) and (15) apply.

(2) Poor when the Reynolds numbers in the tube were in the region 8,000 < Re < 20,000.

No tests were made at Re > 20,000. It may be expected that, for a minimum Reynolds number greater than 20,000, good agreement would be obtained with equation (17) for all turbulent flow. Equations (B26) and (B29) in appendix B may be used to calculate the Reynolds number.

CONCLUSIONS

A method has been presented for determining the averaging characteristics of a total-pressure probe in fluctuating flow, and an experiment has been conducted to check this method. The results and analysis show that

1. The averaging characteristics are a function of the wave shape tested, the gas properties, the total-pressure probe geometry, and the magnitude of the pressures involved.

2. The theoretical and experimental data were in reasonable agreement when the maximum Reynolds number within the probe was less than 8,000.

3. The averaging error is minimized when the inside diameter of the probe entrance tube is made as small as possible, and its length as great as possible, consistent with an acceptable time lag.

Lewis Flight Propulsion Laboratory
    National Advisory Committee for Aeronautics
        Cleveland, Ohio, July 29, 1955

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A $\dfrac{P_m - P_{min}}{P_m}$

a   speed of sound under ambient conditions

C   discharge coefficient

D   inside diameter of total-pressure-probe entrance tube

f   friction factor

G   mass-flow rate per unit area

L   length of entrance tube within total-pressure probe

P   stagnation pressure

P_a time-weighted average of applied pressure

P_m pressure as averaged by total-pressure probe

P_{min} minimum applied stagnation pressure

P_{max} maximum applied stagnation pressure

p_b pressure upstream of equivalent orifice

p_c pressure downstream of equivalent orifice and upstream of tube

p_d pressure downstream of tube

R   gas constant

Re   Reynolds number

T   period of fluctuation

X $\dfrac{P_a - P_m}{P_m - P_{min}}$

α $\dfrac{P_m - P_{min}}{P_{max} - P_{min}}$

β fraction of period that pressure is a maximum

γ ratio of specific heats

δ pressure drop across tube divided by pressure downstream of tube,

$$\delta = \frac{p_c - p_d}{p_d}$$

η probe parameter that involves probe geometry and gas properties,

$$\eta = \frac{D^2 P_{min}}{32\mu LC\sqrt{2R\theta}}$$

θ temperature

μ viscosity

τ_0 time constant of probe and associated system

ψ $\dfrac{P_{max} - P_{min}}{P_{min}}$

Subscripts:

L   flow from left to right (fig. 2(b))

R   flow from right to left (fig. 2(c))

APPENDIX B

DERIVATION OF EQUATIONS

Laminar Flow

Combining equations (7) and (8) yields, for the mass-flow rate through the tube,

$$G = \frac{D^2}{64\mu LR\theta} (p_c^2 - p_d^2) \tag{B1}$$

Since the mass-flow rate through the orifice is equal to the mass-flow rate through the tube at any instant of time, equations (B1), (6), and (14) can be combined to yield

$$\frac{\eta}{2P_{min}} (p_c^2 - p_d^2) = \left[p_c(p_b - p_c) \right]^{1/2} \tag{B2}$$

If the flow through the total-pressure probe is from left to right (fig. 2(b)), then

$$p_{d,L} = P_m = P_{min}(1 + \alpha\psi) \tag{B3}$$

$$p_{b,L} = P_{max} = P_{min}(1 + \psi) \tag{B4}$$

Let

$$\delta_L = \frac{p_{c,L} - P_m}{P_m} \tag{B5}$$

Then

$$p_{c,L} = P_{min}(1 + \alpha\psi)(1 + \delta_L) \tag{B6}$$

Substituting the preceding relations in equation (B2) and grouping terms yield

$$\eta^2 (1 + \alpha\psi)^2 \left(1 + \frac{1}{4} \frac{\delta_L^2}{1 + \delta_L} \right) \delta_L^2 + \delta_L - \frac{(1 - \alpha)\psi}{1 + \alpha\psi} = 0 \tag{B7}$$

Assuming that $\dfrac{1}{4}\dfrac{\delta_L^2}{1+\delta_L}$ is negligibly small and then solving equation (B7) for δ_L yield equation (12).

Combining equations (B1), (B3), and (B6) and then solving for G_L yield

$$G_L = \frac{D^2 P_{min}^2 (1 + \alpha\psi)^2}{64\mu LR\theta} \delta_L(2 + \delta_L) \tag{B8}$$

If the flow through the total-pressure probe is from right to left (fig. 2(c)), then

$$p_{d,R} = P_{min} \tag{B9}$$

$$p_{b,R} = P_m = P_{min}(1 + \alpha\psi) \tag{B10}$$

Let

$$\delta_R = \frac{p_{c,R} - P_{min}}{P_{min}} \tag{B11}$$

Then

$$p_{c,R} = P_{min}(1 + \delta_R) \tag{B12}$$

Substituting the preceding relations in equation (B2) yields

$$\eta^2 \left(1 + \frac{1}{4}\frac{\delta_R^2}{1 + \delta_R}\right)\delta_R^2 + \delta_R - \alpha\psi = 0 \tag{B13}$$

Assuming that $\dfrac{1}{4}\dfrac{\delta_R^2}{1+\delta_R}$ is negligibly small and then solving equation (B13) for δ_R yield equation (13).

Combining equations (B1), (B9), and (B12) and solving for G_R yield

$$G_R = \frac{D^2 P_{min}^2}{64\mu LR\theta} \delta_R(2 + \delta_R) \tag{B14}$$

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Since the net mass flow when the flow is from left to right equals the net mass flow when the flow is from right to left,

$$\beta G_L = (1 - \beta) G_R \tag{B15}$$

Combining equations (B8), (B14), and (B15) and then solving for β yield equation (10).

Consider a different set of parameters η, β, A, and X, where A and X are defined by equations (18) and (19), respectively. Equations (1), (2), (11), (18), and (19) can be combined to yield the following relations:

$$\alpha = \frac{\beta}{1 + X} \tag{B16}$$

$$\psi = \frac{A}{1 - A} \frac{1 + X}{\beta} \tag{B17}$$

Substituting equations (B16) and (B17) in equation (B7), assuming that $\frac{1}{4} \frac{\delta_L^2}{1 + \delta_L}$ is zero, and solving the resulting equation for X yield equation (21). Substituting equations (B16) and (B17) into equation (10) and solving for δ_L yield equation (22). Substituting equations (B16) and (B17) into equation (13) yield equation (23).

If the tube is so long that the pressure drop across the tube is much larger than the pressure drop due to the contraction, η becomes small. Then equation (12) reduces to

$$\lim_{\eta \to 0} \delta_L = \frac{\psi(1 - \alpha)}{1 + \alpha\psi} \tag{B18}$$

and equation (13) reduces to

$$\lim_{\eta \to 0} \delta_R = \alpha\psi \tag{B19}$$

If these expressions for δ_L and δ_R are substituted in equation (10), equation (15) results.

If the tube is so short that the pressure drop across the contraction is much larger than the pressure drop across the tube, η becomes large. By dividing equation (12) by equation (13) there results

$$\frac{\delta_L}{\delta_R} = \frac{1}{(1 + \alpha\psi)^2} \frac{-1 + \left[1 + 4\eta^2(1 + \alpha\psi)\psi(1 - \alpha)\right]^{1/2}}{-1 + (1 + 4\eta^2\alpha\psi)^{1/2}} \tag{B20}$$

and

$$\lim_{\eta \to \infty} \frac{\delta_L}{\delta_R} = \frac{1}{(1 + \alpha\psi)^2} \left[(1 + \alpha\psi) \frac{1 - \alpha}{\alpha}\right]^{1/2} \tag{B21}$$

From equation (12)

$$\lim_{\eta \to \infty} (2 + \delta_L) = 2 \tag{B22}$$

From equation (13)

$$\lim_{\eta \to \infty} (2 + \delta_R) = 2 \tag{B23}$$

If equations (B21), (B22), and (B23) are substituted in equation (10), equation (16) results.

By definition, Reynolds number is given by

$$\text{Re} = \frac{GD}{\mu} \tag{B24}$$

For the case in which the flow is from right to left,

$$\text{Re}_R = \frac{G_R D}{\mu} \tag{B25}$$

If equations (13), (14), (B14), and (B25) are combined, the following Reynolds number expression for flow from right to left results:

$$\text{Re}_R = 32C \frac{2L}{D}\alpha\psi \left(\frac{4\eta^2 - 1 + \sqrt{1 + 4\eta^2\alpha\psi}}{1 + \sqrt{1 + 4\eta^2\alpha\psi}}\right) \tag{B26}$$

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Note that

$$\alpha\psi = \frac{A}{1 - A} \tag{B27}$$

If both sides of equation (B15) are multiplied by D/μ, the equation becomes

$$\beta \frac{G_L D}{\mu} = (1 - \beta) \frac{G_R D}{\mu} \tag{B28}$$

Since $G_L D/\mu$ equals Re_L and $G_R D/\mu$ equals Re_R, equation (B28) can be solved for Re_L and the following relation results:

$$Re_L = \left(\frac{1 - \beta}{\beta}\right) Re_R \tag{B29}$$

Turbulent Flow

Only that case will be considered where the total-pressure tube is so long that the pressure drop due to the tube is much larger than the pressure drop due to the contraction. If equations (7) and (9) are combined, the mass-flow rate through the tube becomes

$$G = \left(\frac{D^{1.2}}{0.18\mu^{0.2}LR\theta}\right)^{5/9} (p_c^2 - p_d^2)^{5/9} \tag{B30}$$

For the flow from left to right (fig. 2(b)),

$$p_{c,L} = P_{max} = P_{min}(1 + \psi) \tag{B31}$$

$$p_{d,L} = P_m = P_{min}(1 + \alpha\psi) \tag{B32}$$

Substituting these expressions for p_c and p_d in equation (B30) yields

$$G_L = \left(\frac{D^{1.2}P_{min}^2}{0.18\mu^{0.2}LR\theta}\right)^{5/9} \left[\psi(1 - \alpha)(2 + \psi + \alpha\psi)\right]^{5/9} \tag{B33}$$

For the flow from right to left (fig. 2(c)),

$$p_{c,R} = P_m = P_{min}(1 + \alpha\psi) \tag{B34}$$

$$p_{d,R} = P_{min} \tag{B35}$$

Substituting these expressions for p_c and p_d in equation (B30) yields

$$G_R = \left(\frac{D^{1.2}P_{min}^2}{0.18\mu^{0.2}LR\theta}\right)^{5/9} \cdot \left[\alpha\psi(2 + \alpha\psi)\right]^{5/9} \tag{B36}$$

If equations (B15), (B33), and (B36) are combined and then solved for β, equation (17) results.

APPENDIX C

SAMPLE CALCULATION

A sample calculation is the measurement of the time-average stagnation pressure in an air stream whose stagnation pressure fluctuates. An independent measurement indicates that the frequency of the fluctuation is 200 cps and the wave form can be approximated by a value of β of 0.75. Other data are

Indicated average total pressure, P_m, in. Hg abs 40.2

Minimum stagnation pressure, P_{min}, in. Hg abs 25.1

Temperature, $^{\circ}R$ 530

Probe dimensions:

    Entrance-tube I.D., D, in. 0.020

    Entrance-tube length, L, in. 2.0

    Entrance tube opens into 0.1-in.-I.D. probe which is connected to a manometer by 0.1-in.-I.D. tubing.

The time constant τ_0 of the probe and manometer connections was determined to be approximately 0.1 second.

Since a = 13,000 inches per second, L/a = 0.0002 second, τ_0 = 0.1 second, and T = 0.005 second, the inequality $2\pi\tau_0 \gg T \gg L/a$ is satisfied. From equation (14), η = 2.5; from equation (18), A = 0.38; from equation (B27), $\alpha\psi$ = 0.61; from equation (B26), Re_R = 5400; and from equation (B29), Re_L = 1800. Since the maximum Reynolds number is less than 8,000, it is permissible to use table I to find the value of X. By interpolation, this value is found to be -0.13.

From equation (20), P_a = 38.2 inches of mercury absolute. Thus, the time-average pressure is 5 percent lower than the indicated pressure.

REFERENCE

1. Nesbitt, M. V.: The Measurement of True Mean Pressures and Mach Numbers in Oscillatory Flow. Memo. No. M.180, British N.G.T.E., Mar. 1953.

TABLE I. - VALUE OF X AS FUNCTION OF β, η, AND A

$$[P_a = P_m \ (1 + AX)]$$

(a) A = 0.005.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.021	0.012	0.111	0.462	1.393	3.968	4.560	7.180
.2	-.009	.002	.035	.157	.470	.997	1.522	2.384
.3	-.004	0	.012	.060	.182	.393	.584	.924
.4	-.003	0	.003	.016	.057	.124	.192	.296
.5	-.002	-.002	-.002	-.002	-.002	-.002	-.002	-.002
.6	-.001	-.002	-.006	-.010	-.028	-.060	-.078	-.135
.7	-.001	-.002	-.005	-.011	-.036	-.069	-.114	-.172
.8	-.001	-.001	-.003	-.013	-.030	-.068	-.095	-.150
.9	0	-.001	-.001	-.009	-.017	-.042	-.057	-.090
1.0	0	0	0	0	0	0	0	0

(b) A = 0.01.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.042	0.024	0.211	0.786	2.050	3.699	5.090	7.120
.2	-.020	.003	.070	.266	.699	1.251	1.710	2.370
.3	-.011	-.002	.025	.101	.269	.487	.662	.914
.4	-.007	-.004	.005	.029	.084	.151	.207	.290
.5	-.004	-.004	-.004	-.004	-.005	-.005	-.005	-.005
.6	-.003	-.006	-.007	-.020	-.042	-.072	-.104	-.136
.7	-.002	-.005	-.009	-.022	-.054	-.094	-.123	-.172
.8	-.001	-.004	-.006	-.018	-.049	-.080	-.113	-.151
.9	-.001	-.001	-.005	-.013	-.029	-.049	-.064	-.089
1.0	0	0	0	0	0	0	0	0

TABLE I. - Continued. VALUE OF X AS FUNCTION OF β, η, AND A

$$[P_a = P_m \ (1 + AX)]$$

(c) A = 0.02.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.077	0.043	0.365	1.220	2.710	[a]4.310	[a]5.545	[a]7.050
.2	-.038	.006	.122	.427	.940	1.464	1.875	2.335
.3	-.022	-.002	.042	.162	.366	.566	.735	.900
.4	-.015	-.006	.008	.046	.112	.178	.229	.282
.5	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010
.6	-.007	-.009	-.014	-.034	-.064	-.094	-.106	-.139
.7	-.004	-.008	-.016	-.038	-.076	-.116	-.141	-.174
.8	-.002	-.006	-.013	-.034	-.068	-.098	-.124	-.156
.9	-.001	-.003	-.007	-.019	-.039	-.060	-.070	-.089
1.0	0	0	0	0	0	0	0	0

(d) A = 0.04.

0.1	-0.145	0.078	0.573	1.690	[a]3.230	[a]4.640	[a]5.630	[a]6.875
.2	-.070	.014	.199	.607	1.142	1.604	1.908	2.275
.3	-.043	-.008	.067	.231	.440	.620	.733	.795
.4	-.028	-.016	.008	.062	.130	.184	.223	.264
.5	-.019	-.019	-.019	-.020	-.020	-.020	-.020	-.020
.6	-.013	-.017	-.030	-.054	-.085	-.110	-.124	-.144
.7	-.008	-.014	-.030	-.061	-.102	-.133	-.152	-.177
.8	-.005	-.010	-.023	-.051	-.085	-.113	-.132	-.152
.9	-.002	-.005	-.013	-.030	-.050	-.066	-.077	-.089
1.0	0	0	0	0	0	0	0	0

[a]Value of X for which theory breaks down because of limitation 2 in DISCUSSION AND RESULTS.

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TABLE I. - Continued. VALUE OF X AS FUNCTION OF β, η, AND A

$$[P_a = P_m (1 + AX)]$$

(e) A = 0.07.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.201	0.108	0.756	1.990	[a]3.460	[a]4.710	[a]5.560	--
.2	-.111	.015	.283	.733	1.242	1.635	1.887	[a]2.180
.3	-.071	-.018	.090	.276	.474	.621	.712	.820
.4	-.047	-.030	.056	.067	.131	.176	.204	.237
.5	-.033	-.033	-.033	-.033	-.033	-.034	-.035	-.035
.6	-.022	-.031	-.047	-.077	-.104	-.125	-.137	-.152
.7	-.015	-.025	-.047	-.084	-.120	-.147	-.164	-.180
.8	-.009	-.018	-.037	-.069	-.101	-.124	-.139	-.154
.9	-.004	-.010	-.021	-.040	-.059	-.072	-.080	-.090
1.0	0	0	0	0	0	0	0	0

(f) A = 0.1.

β	0	1	2	4	8	16	32	∞
0.1	-0.254	0.133	0.866	[a]2.114	[a]3.509	--	--	--
.2	-.146	.092	.319	.790	1.266	[a]1.614	[a]1.827	[a]2.080
.3	-.096	-.025	.101	.293	.477	.608	.685	.770
.4	-.065	-.042	.001	.064	.122	.070	.186	.210
.5	-.046	-.046	-.046	-.046	-.047	-.049	-.050	-.050
.6	-.032	-.043	-.063	-.092	-.119	-.137	-.148	-.160
.7	-.021	-.036	-.062	-.099	-.134	-.156	-.171	-.184
.8	-.012	-.026	-.048	-.082	-.112	-.131	-.144	-.155
.9	-.006	-.014	-.027	-.047	-.065	-.076	-.083	-.090
1.0	0	0	0	0	0	0	0	0

[a]Values of X for which theory breaks down because of limitation 2 in DISCUSSION AND RESULTS.

TABLE I. - Continued. VALUE OF X   AS FUNCTION OF β, η, AND   A

$$[P_a = P_m (1 + AX)]$$

(g) A = 0.2.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.370	0.194	[a]1.020	--	--	--	--	--
.2	-.237	.023	0.375	[a]0.810	[a]1.183	[a]1.437	[a]1.588	[a]1.760
.3	-.166	-.048	.103	.279	.404	.504	.554	.606
.4	-.118	-.077	-.025	.028	.068	.092	.106	.120
.5	-.085	-.085	-.086	-.088	-.093	-.097	-.100	-.100
.6	-.060	-.080	-.106	-.134	-.157	-.171	-.179	-.186
.7	-.039	-.067	-.100	-.136	-.162	-.180	-.187	-.197
.8	-.024	-.048	-.076	-.104	-.132	-.145	-.152	-.160
.9	-.011	-.026	-.043	-.062	-.074	-.083	-.086	-.091
1.0	0	0	0	0	0	0	0	0

(h) A = 0.3.

β	0	1	2	4	8	16	32	∞
0.1	-0.445	[a]0.249	[a]1.105	--	--	--	--	--
.2	-.304	.029	.379	[a]0.758	[a]1.030	[a]1.216	--	--
.3	-.219	-.066	.084	.224	.320	.378	[a]0.409	[a]0.442
.4	-.161	-.110	-.059	-.019	.038	.016	.025	.030
.5	-.118	-.119	-.123	-.130	-.138	-.140	-.146	-.150
.6	-.085	-.113	-.141	-.169	-.188	-.200	-.206	-.213
.7	-.058	-.095	-.130	-.162	-.184	-.196	-.202	-.210
.8	-.035	-.069	-.099	-.126	-.144	-.155	-.160	-.165
.9	-.016	-.037	-.055	-.071	-.081	-.086	-.086	-.092
1.0	0	0	0	0	0	0	0	0

[a]Value of X for which theory breaks down because of limitation 2 in
  DISCUSSION AND RESULTS.

TABLE I. - Concluded. VALUE OF X AS FUNCTION OF β, η, AND A

$$[P_a = P_m (1 + AX)]$$

(i) A = 0.4.

β	η							
	0	1	2	4	8	16	32	∞
0.1	-0.500	[a]0.315	--	--	--	--	--	--
.2	-.356	.040	[a]0.361	[a]0.651	--	--	--	--
.3	-.265	-.080	.051	.153	[a]0.212	[a]0.247	[a]0.263	[a]0.270
.4	-.200	-.133	-.096	-.073	-.065	-.061	-.059	-.060
.5	-.149	-.150	-.159	-.172	-.183	-.191	-.194	-.200
.6	-.109	-.143	-.172	-.200	-.221	-.227	-.233	-.240
.7	-.075	-.120	-.155	-.184	-.202	-.213	-.218	-.223
.8	-.046	-.088	-.117	-.140	-.154	-.162	-.166	-.170
.9	-.021	-.046	-.064	-.077	-.085	-.089	-.091	-.094
1.0	0	0	0	0	0	0	0	0

(j) A = 0.5.

β	0	1	2	4	8	16	32	∞
0.1	-0.543	[a]0.386	--	--	--	--	--	--
.2	-.400	.058	[a]0.331	--	--	--	--	--
.3	-.304	-.090	.010	[a]0.070	[a]0.096	[a]0.107	[a]0.111	[a]0.082
.4	-.233	-.157	-.135	-.133	-.139	-.143	-.147	-.150
.5	-.176	-.176	-.194	-.214	-.229	-.239	-.246	-.250
.6	-.131	-.170	-.202	-.229	-.247	-.256	-.262	-.267
.7	-.091	-.145	-.178	-.204	-.219	-.227	-.232	-.236
.8	-.056	-.116	-.133	-.142	-.163	-.169	-.172	-.175
.9	-.027	-.057	-.072	-.083	-.088	-.091	-.093	-.094
1.0	0	0	0	0	0	0	0	0

[a]Value of X for which theory breaks down because of limitation 2 in DISCUSSION AND RESULTS.

TABLE II. - PROBE CHARACTERISTICS

| Probe	D, in.	L, in.	η, air under standard conditions	Re_R for $\alpha\psi$ of 0.05	Re_R for $\alpha\psi$ of 0.8	Agreement between theory and experiment
1	0.012	12	0.18	52	1,100	Good
2	.012	2	1.1	310	4,000	Good
3	.017	2	2.2	760	6,600	Good
4	.016	1.06	3.5	910	6,200	Good
5	.017	.38	12	1600	7,400	Good
6	.020	.010	400	--	----	Good
7	.029	6.5	1.9	1100	11,000	Good
8	.029	3.3	3.8	1800	12,000	Good
9	.029	1.0	12	2400	12,000	Good
10	.032	12	1.3	950	11,000	Good for $\psi < 0.4$ Poor for $\psi > 0.4$
11	.047	15	2.2	2100	18,000	Poor
12	.047	9	3.7	2900	19,000	Poor
13	.047	1.6	21	4500	20,000	Poor
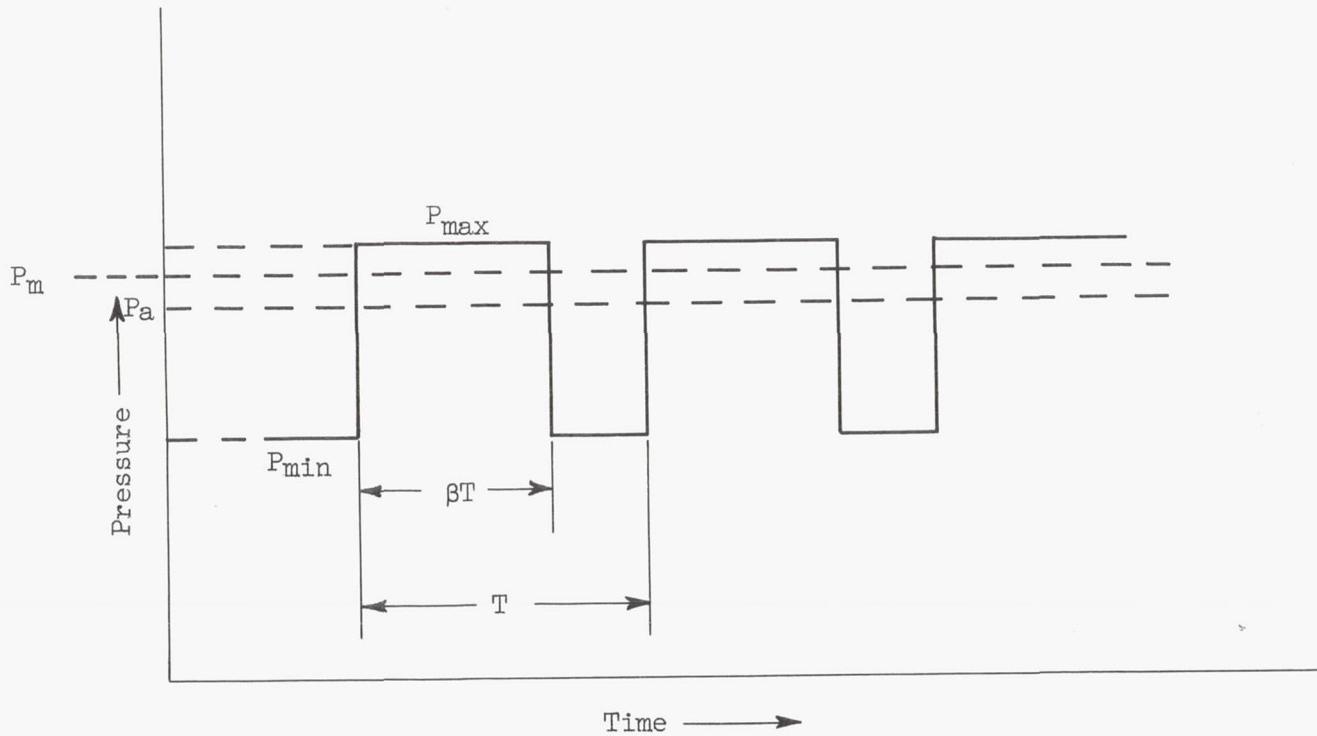
Figure 1. - Wave form of applied pressure. P_a, time-weighted average of applied pressure; P_m, average pressure indicated by total-pressure probe; P_{max}, maximum pressure; P_{min}, minimum pressure; T, period of fluctuation; β, fraction of period that pressure is a maximum.
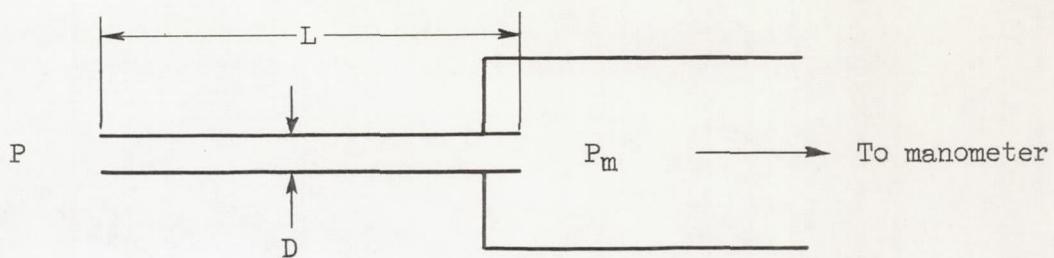
(a) Probe dimensions.
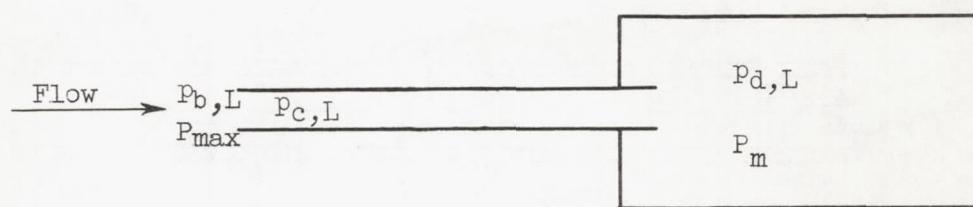
(b) Flow from left to right.
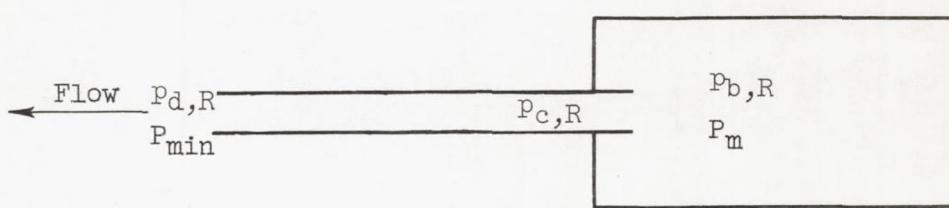
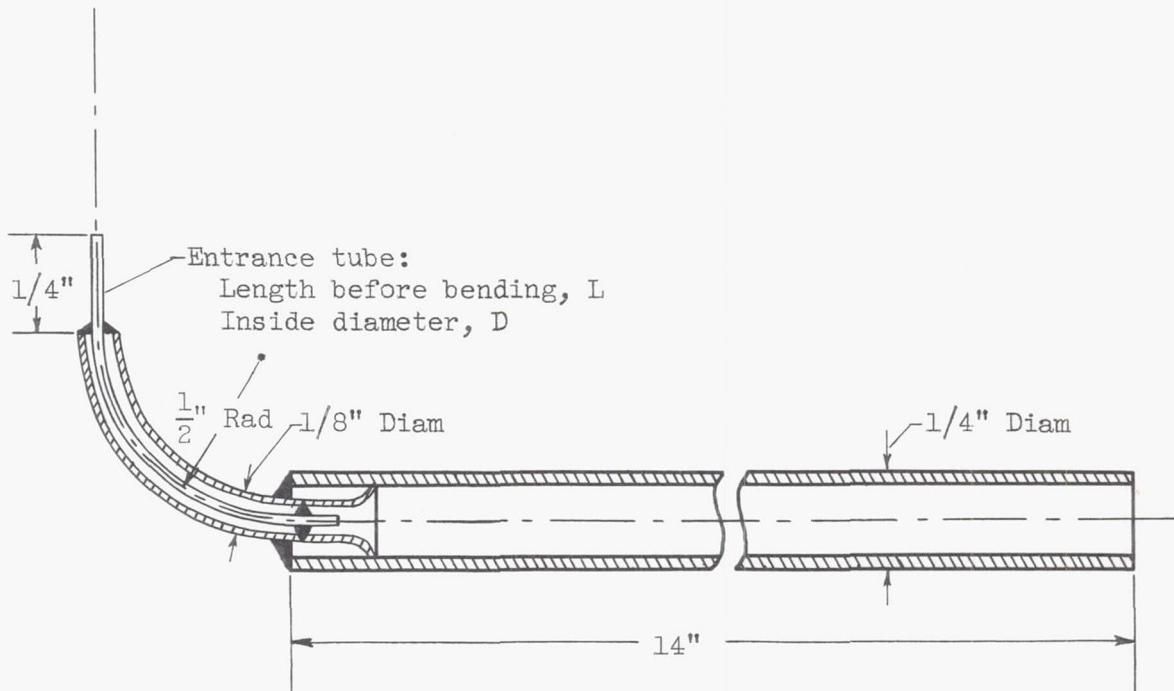(c) Flow from right to left.

Figure 2. - Simplified sketch of probe.

Figure 3. - Schematic diagram of total-pressure probe. Cross section of all probes except 11, 12, and 13 for which the 1/8-inch-diameter tube is omitted.
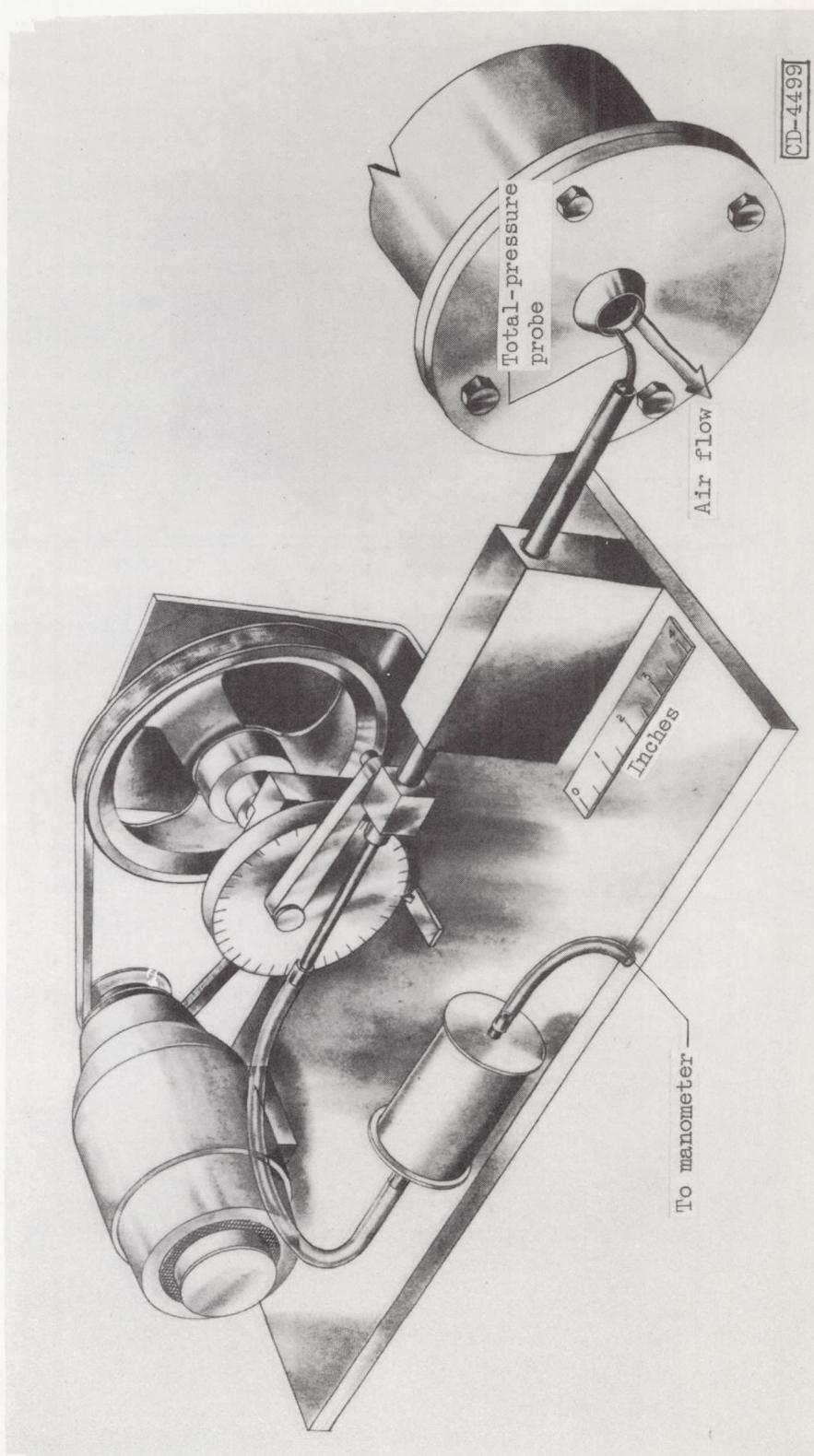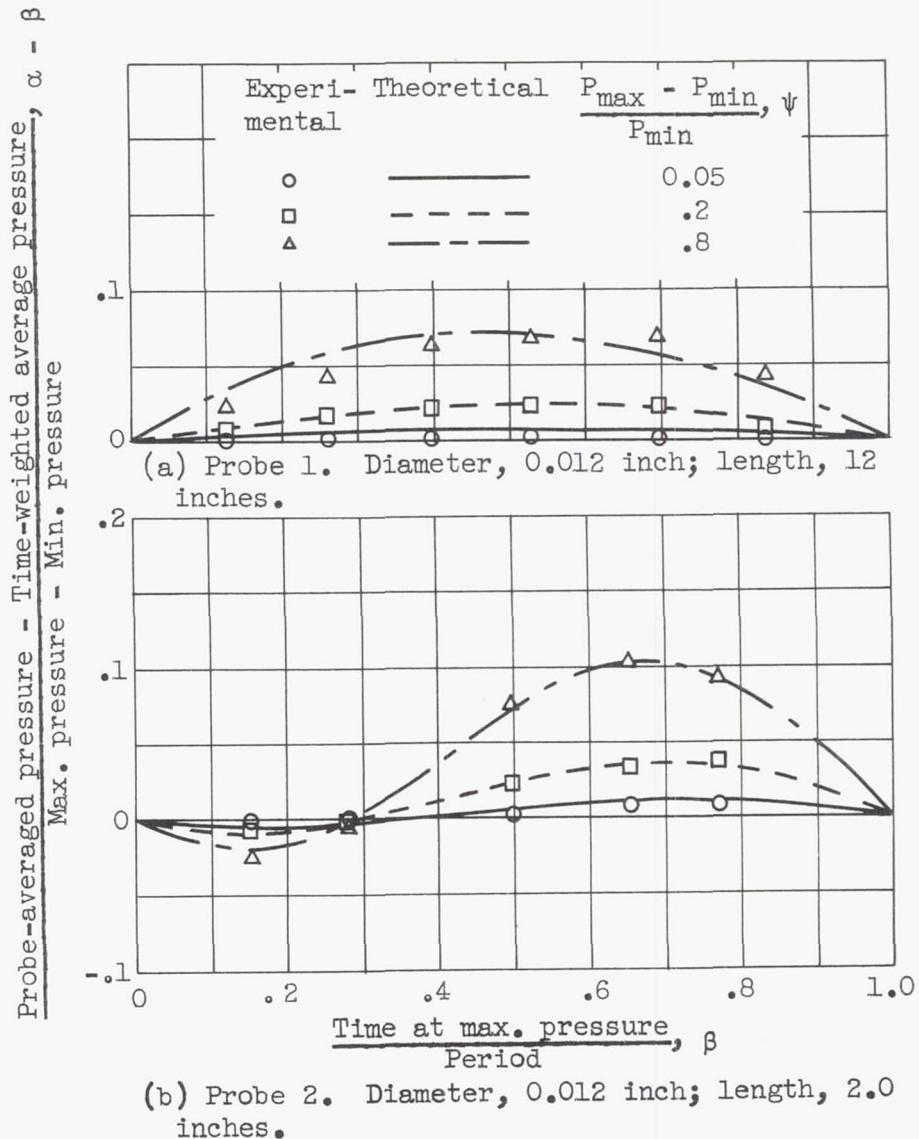
Figure 4. - Test apparatus.

(a) Probe 1. Diameter, 0.012 inch; length, 12 inches.

(b) Probe 2. Diameter, 0.012 inch; length, 2.0 inches.

Figure 5. - Comparison of calculated and experimental probe error.

(c) Probe 5. Diameter, 0.017 inch; length, 0.38 inch.
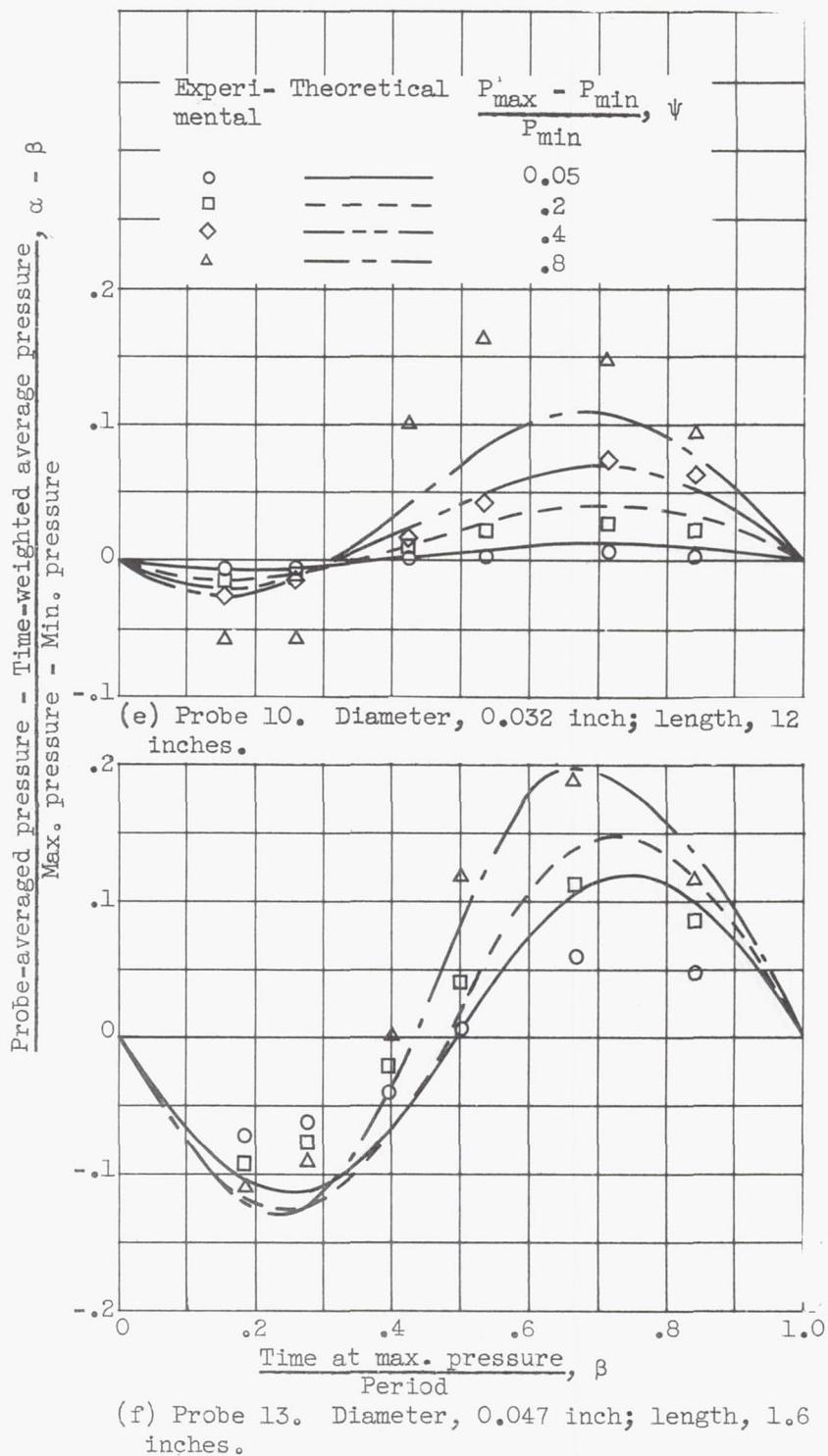
(d) Probe 6. Diameter, 0.020 inch; length, 0.010 inch.
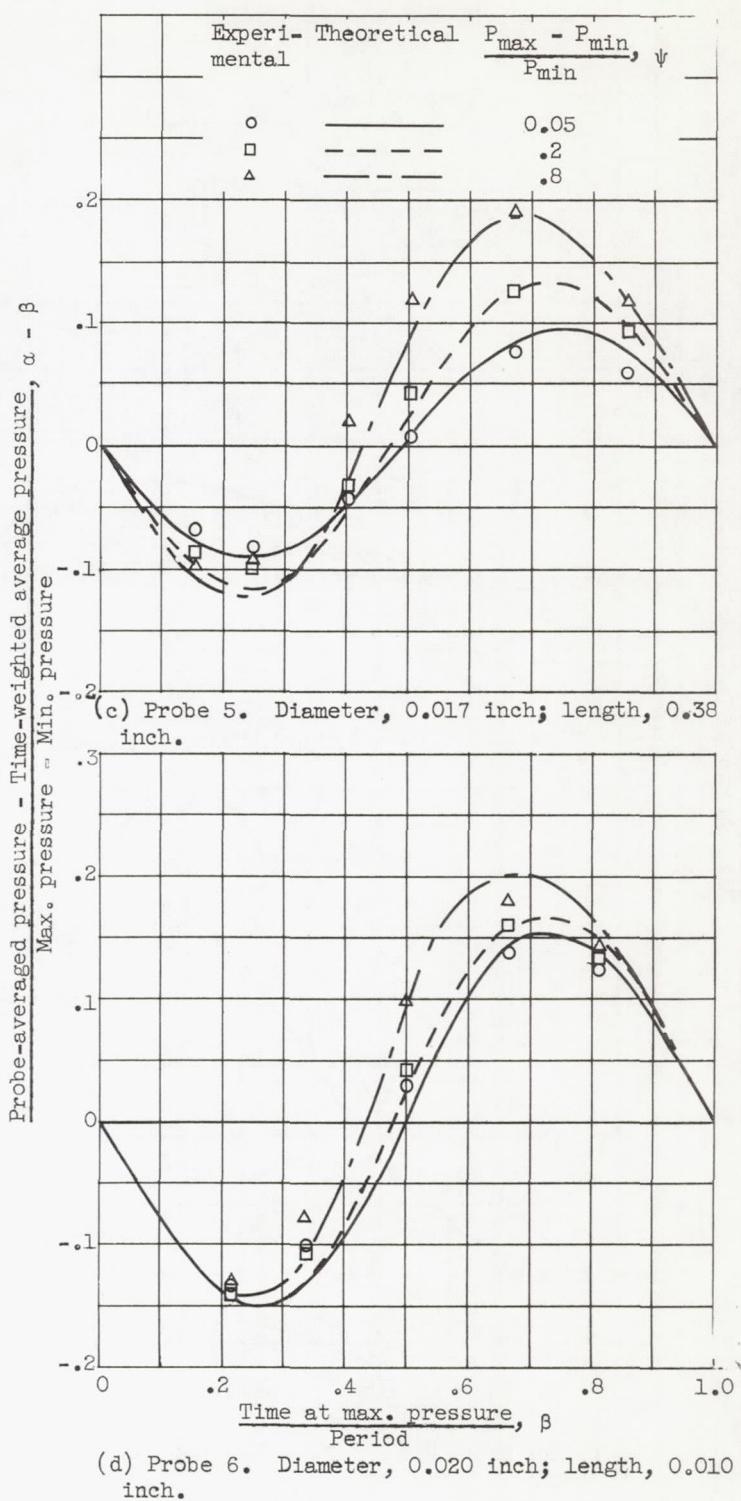
Figure 5. - Continued. Comparison of calculated and experimental probe error.

(e) Probe 10. Diameter, 0.032 inch; length, 12 inches.

(f) Probe 13. Diameter, 0.047 inch; length, 1.6 inches.

Figure 5. - Concluded. Comparison of calculated and experimental probe error.